# Demonstrator # 7

## Angular amplitude of a swing’s oscillation

### TEACHER NOTES

**Activity title:**

Determining the angular amplitude of oscillation of a swing, trapeze sport, etc.

**Subject:**

Physics - Class XI

**Student age:**

16-18 years

**Estimated duration:**

2x50 minutes (50 minutes, for data collecting, 50 minutes for data processing)

### Science content

- The Principles of Newtonian dynamics;
- The mechanical laws concerning: motion, speed, the acceleration of harmonic oscillations;
- The InLOT System;
- The mechanical oscillations;
- The elastic pendulum.

### Learning objectives

Lesson creatively exploit knowledge is valuable because Newtonian dynamics, trigonometry, practical skills through non-formal learning contexts applicability, such as sport.

**SPECIFIC COMPETENCIES:** At the end of this lesson students will be able to:

- to apply the IVth of Newtonian dynamics and to enhance understanding of the principles of Newtonian dynamics
- to apply the oscillatory motion laws and to strengthen their understanding
- to valorify the knowledge of trigonometry
- to use creatively INLOT system in applied contexts
- to explore the physics reality by testing AM on trapeze, swing

### Inquiry-based character

The student will enhance their work skills specific scientific investigation and discovery
activities geared for this type of learning:

1. **Identify Questions for Scientific Investigations**
   - Identify testable questions
   - Refine/refocus ill-defined questions
   - Formulate hypotheses

2. **Design Scientific Investigations**
   - Design investigations to test a hypothesis
   - Identify independent variables, dependent variables, and variables that need to be controlled
   - Operationally define variables based on observable characteristics
   - Identify flaws in investigative design
   - Utilize safe procedures
   - Conduct multiple trials

3. **Use Tools and Techniques to Gather Data**
   - Gather data by using appropriate tools and techniques
   - Measure using standardized units of measure
   - Compare, group, and/or order objects by characteristics
   - Construct and/or use classification systems
   - Use consistency and precision in data collection
   - Describe an object in relation to another object (e.g., its position, motion, direction, symmetry, spatial arrangement, or shape)

4. **Analyze and Describe Data**
   - Differentiate explanation from description
   - Construct and use graphical representations
   - Identify patterns and relationships of variables in data
   - Use mathematic skills to analyze and/or interpret data

5. **Explain Results and Draw Conclusions**
   - Differentiate observation from inference
   - Propose an explanation based on observation
   - Use evidence to make inferences and/or predict trends
   - Form a logical explanation about the cause-and-effect relationships in data from an experiment

6. **Recognize Alternative Explanations and Predictions**
   - Consider alternate explanations
   - Identify faulty reasoning not supported by data

7. **Communicate Scientific Procedures and Explanations**
   - Communicate experimental and/or research methods and procedures
   - Use evidence and observations to explain and communicate results
   - Communicate knowledge gained from an investigation orally and through written reports, incorporating drawings, diagrams, or graphs where appropriate

**Applied technology (if any)**

In order to do so the KLiC project uses an innovative sensor data collection tool, namely the InLOT system (www.inlot.eu) that consists of the following modules:

- **SensVest** - a vest, equipped with various sensors, designed to carry components that
measure and transmit physiological data to the base station.

- **Leg and Arm Accelerometer** - small devices attached to the leg and/or arm that enable the 3-D measurement of the acceleration for the leg and/or arm.
- **Ball Accelerometer** - a ball that has embedded an accelerometer measuring three dimensions and a communication unit that enables the transmission of data packets to the base.
- **Base Station** - responsible for the collection of all transmitted data
- **User Interface Software** - user friendly interface, designed with a pedagogical frame of mind, that enables the process of data and actions such as plotting data on a graph or creating a mathematical model to fit the data.

User details can be found in Annex 2.1.

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<tr>
<th>Materials needed</th>
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<tr>
<td>- InLOT system</td>
</tr>
<tr>
<td>- PC</td>
</tr>
<tr>
<td>- Physical kit: mechanical oscillations</td>
</tr>
<tr>
<td>- Worksheet (Annexes 2.1, 2.2 and 2.3)</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Discussion guide</th>
</tr>
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<tbody>
<tr>
<td><strong>Anticipation:</strong> Unit summary: Mechanical Oscillations</td>
</tr>
</tbody>
</table>

**Essential Question:** *How physics helps us to better understand the surrounding world?*

**Before a project approach**

Before using a project approach, the high school students will review the principles of Newtonian dynamics, will discuss techniques for working with INLOT system, then write an essay about the use of physical knowledge in sports. Essays will be between three and five pages and will be noted. Essays will be evaluated in terms of Newtonian dynamics harnessing knowledge about techniques for working with INLOT system discussed above.

**After a project approach**

After the scenario proposed sequence no.2 has been completed, indicated that students apply the theme and new skills to the situations described by their essays. Students will be invited to explore the questions: a) *How physics helps us to better understand the surrounding world?* and b) *How that gives us the performance perspective?*. Students will analyze how science and technology in performance are mutually supportive and not just athletes

**Building knowledge**

**Teaching strategy**

- The teacher monitors and advises business groups, provides support points, support students in their approach.
- Use project method
- Integrate knowledge and skills achieved an adequate framework for reflection.

**Reflection / Consolidation**

**Evaluation method:** gallery tour

**Assessment**

- summative
- formative
Annex 2.1

Using accelerometer

Reference directions of accelerometer

What accelerometer (AM) measures?
- The frames of reference in which the experiments are conducted are non-inertial, so it is necessary to simplify the model; therefore we encourage the selection of appropriate experimental contexts secondary level approach.
- It appears that AM measures, momentary, relative acceleration in non-inertial frames of reference. Generally, according to kinematics in non-inertial frames of reference:

\[ \ddot{a}_{\text{rel}} = \ddot{a}_{\text{abs}} - (\ddot{a}_{\text{cor}} + \ddot{a}_{\text{transp}}) \]  
\[ m \cdot \ddot{a}_{\text{rel}} = m \cdot \ddot{a}_{\text{abs}} - m \cdot (\ddot{a}_{\text{cor}} + \ddot{a}_{\text{transp}}) \]  
\[ m \cdot \ddot{a}_{\text{rel}} = \vec{F} + \vec{F}_{c} \]  

-Accelerometer (AM) measures the difference between the momentary gravitational component (reference direction Ox of AM), plus centrifugal momentary acceleration (if a change of direction of motion) and momentary acceleration of movement of AM in that direction.

\[ a_{s} = g_{s} + a_{efx} - a_{m} \]  

Where:  
- \( a_{s} \) is the value measured on test direction (relative acceleration)  
- \( g_{s} \) is the component of gravity acceleration on test direction

1. where \( \vec{F}_{c} \) is supplementary forces.

Particularly, there are situations (eg, a ball suspended at rest relative to the earth, but relative to a man sitting on a rotating wheel, the ball appears to be in rotation), where it may happen that the body viewed from S does not any force, but still to see him moving accelerated relative to S' due to supplementary force, \( \vec{F}_{c} \):

\[ a_{\text{abs}} = 0 \Rightarrow \vec{F} = 0 \rightarrow \vec{F}_{c} = m \cdot \ddot{a}_{\text{rel}} \]  

An important class of reference frames is the object's own frame or frame-related rigid object moving uniformly force from their frame (eg the man and the object (= S') are resting on the rotating disc, and the object is caught in a spring). In such frames the object is evident in the rest (\( \ddot{a}_{\text{rel}} = 0 \)), although there is a real force \( \vec{F} \). In this case: \( \vec{F} + \vec{F}_{c} = 0 \rightarrow \vec{F}_{c} = m \cdot \ddot{a}_{\text{rel}} \). That supplementary force is equal but opposite to the real force, so it is equivalent to the Newtonian inertial force.

Supplementary forces are fictitious forces that should be added to the real forces to ensure the validity of the II\textsuperscript{nd} principle of Newtonian mechanics in non-inertial frames. These are not forces of interaction, we can show the body that produces them, so it doesn’t applies the III\textsuperscript{rd} principle of Newtonian mechanics.
- $a_{cf}$ is the component of centrifugal acceleration on test direction
- $a_{ma}$ is the acceleration of movement (accelerometer and body together) on test direction (acceleration of transport).

\[
\begin{align*}
  a_x &= g_y + a_{cfy} - a_{my} \\
  a_z &= g_z + a_{cfz} - a_{mz}
\end{align*}
\] (0.6.)

If the motion is made on certain direction, relatively to the reference directions of AM, then the previous relations are wrote on each component of the acceleration measured by accelerometer ($\neq 0$).

All measured values are fractions of g (gravity acceleration), expressed relative to the value of g for which was calibrated AM.

**Cases:**

I. $a_{ma} = 0$ (AM is at rest, set on the object whose motion is studied, or in rectilinear and uniform motion on test axis, chosen as the Ox axis)

\[ a_x = g_x + a_{cfx} \] (0.8.)

- More if $a_{cfx} = 0$

\[ a_x = g_x \] (0.9.)

II. $g_x = 0$ (the test axis is in a perpendicular plane on vertical)

\[ a_x = a_{cfx} - a_{mx} \] (0.10.)

- In addition if $a_{cfx} = 0$

\[ a_x = -a_{mx} \] (0.11.)

This is the method of determining the acceleration of motion of AM/the object bounded on AM.

**What we can measure with the accelerometer in the laboratory / practical applications?**

- **Angles:** AM in resting, sat alongside a surface makes an angle $\alpha$ with the vertical;

\[ a_x = g \cdot \sin \alpha \Rightarrow \alpha = \arcsin \frac{a_x}{g} \] (0.12.)

- Acceleration of translational motion on:
  - Axis in the horizontal plane regardless of the gravity component
  - Axis of the other plane, but taking into account the gravity component

- Acceleration of complex motion (rotation and translation)
Annex 2.2

ASSESSMENT TOOLS

*Scores for project evaluation*

<table>
<thead>
<tr>
<th>1 = Criterion is not fulfilled</th>
<th>3 = Criterion is fulfilled in good measure</th>
</tr>
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<tbody>
<tr>
<td>2 = Criterion is met only slightly</td>
<td>4 = The criterion is fully met</td>
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1. All team members undertake collaborative activities by completing the steps in processing aid given to them and collect data for one of the roles within the team
   1 2 3 4

2. Each member fulfills the role it has in the team. Team members’ work together to achieve a quality presentation
   1 2 3 4

3. Presentation made meet the recommended structure.
   1 2 3 4

4. Explanation contained in the presentation is enlightening to the public
   1 2 3 4

5. Project presentation is eloquent and enlightening for the audience participating.
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6. The manner of presentation is attractive and involving public
   1 2 3 4

7. Team members are open to public questions and formulate answers all questions pertinent to public
   1 2 3 4

8. Introducing the team roles demonstrates that members are knowledgeable in all fields covered by the project.
   1 2 3 4

9. Team members speak out loud, communicates a very clear presentation of content, and establish eye contact with audience.
   1 2 3 4

10. Team members provide additional explanations to the public request, using the flip chart
    1 2 3 4

**Completion:**

**Note:** The lesson is built valuing prior knowledge acquired in different learning contexts and integrates communication skills, collaboration skills, investigation, practical skills, but also interpersonal and social skills, artistic skills and expression.
Annex 2.3
AUXILIARY FOR TEACHING
2.1. Kicking life into Classroom: Determining the angular amplitude of oscillation of a swing, trapeze sport, etc.

2.2. Into Lab with InLOT

Study gravitational pendulum, practical application for determining the angular amplitude of oscillation of a swing, trapeze sport, etc., in approximation of the harmonic oscillations.

<table>
<thead>
<tr>
<th>Modeling physical phenomena: harmonic oscillations of the gravitational pendulum</th>
<th>Principle method</th>
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<tbody>
<tr>
<td>Gravitational pendulum consists of an object (a material point) with mass m suspended by a nonexpendable wire, with length R (fig.2.1.). If the system is out of equilibrium position (vertical position of the wire) and then left free, it will make oscillatory movements around the equilibrium position. ( \alpha ) angle which between wire at a time and the vertical, is called the angular displacement and ( \alpha_0 ) is angular amplitude. Figure 2.1. shows that the force who tends to restore the system in equilibrium position is ( F = G_i = m \cdot g \cdot \sin \alpha ).</td>
<td>2.II.A. The determination of gravity acceleration of gravitational pendulum method</td>
</tr>
</tbody>
</table>

1. For \( \alpha = 0 \) follows \( a = 0 \); collecting data by InLOT follows \( \Delta t \) between two moments with \( a = 0 \), then follows \( T/2 \) (half period); follows \( T = 2 \Delta t \).
2. \( \alpha = \pm \alpha_0 \)
3. \( a(2\pi) = \omega_0^2 \), and in harmonic oscillations approximation, means \( \alpha_0 < 5^\circ \), follows \( \sin \alpha \cong \alpha \) (radians), we can take into account that the object motion is oriented along Ox axis, and the force which restore the equilibrium is:

\[
F = G_i = -mg \alpha = -mg \frac{x}{R} = -kx
\]

(2.7)

\[
k = m \omega_0^2 = m \frac{g}{R}
\]

(2.8)

\[
\omega_0 = \sqrt{\frac{g}{R}} \Rightarrow \omega_0^2 = \frac{g}{R} \Rightarrow \left( \frac{2\pi}{T} \right)^2 = \frac{g}{R}
\]

(2.9)

\[
\Rightarrow g_{\text{dynamic}} = R \left( \frac{2\pi}{T} \right)^2
\]

(2.10)

2.10. expression allows us to calculate the gravitational acceleration with the...
In laboratory reference frame, on oscillation direction the equations of motion, of velocity and acceleration are:

\[ x(t) = A \sin (\omega_0 t - \pi/2), \quad t=0, \quad x=-A \]  
\[ v(t) = \omega_0 A \cos (\omega_0 t - \pi/2) \]  
\[ a(t) = -\omega_0^2 A \sin (\omega_0 t - \pi/2) \]

Note: \( \omega_0 t - \pi/2 = \alpha(t) \), \( A=R \sin \alpha_0 \)

Then, angular displacement follows:

\[ \alpha(t) = \alpha_0 \sin (\omega_0 t - \pi/2), \quad t=0, \quad \alpha=-\alpha_0 \]  
\[ \dot{\alpha}(t) = \omega_0 \alpha_0 \cos (\omega_0 t - \pi/2) \]  
\[ \ddot{\alpha}(t) = -\omega_0^2 \alpha_0 \sin (\omega_0 t - \pi/2) \]

The equations that are adapted of the harmonic oscillations \( (\alpha_0<5^\circ) \) of the gravitational pendulum.

2.II.B The determination of amplitude and angular amplitude of the harmonical oscillations

\[ a(2k\pi) = g \frac{A}{R} \Rightarrow A=R \frac{a(2k\pi)}{g} \]  
\[ \text{on the other hand:} \]
\[ -a(2k\pi) = a_{\text{max}} \text{ (from measurements)} \]  
\[ -a((2k+1)\pi) = a_{\text{min}} \]  
\[ \Lambda=R \frac{a_{\text{max}}}{g} \Leftrightarrow \alpha_0 = \frac{a_{\text{max}}}{g} \]
STUDENT WORKSHEET

Activity title:
Determining the angular amplitude of oscillation of a swing, trapeze sport, etc.

Introduction

Curriculum-Framing Questions
Essential Question
How would the universe appear without regular phenomena?

Unit Questions.
At what extent the laws of mechanics which are already known can be applied to periodic phenomena?
What immediate applications do you see for the study of periodic phenomena in nature?

Questions of content
What periodical mechanical phenomena can we identify in the nature?
What physical quantities are characteristic for the oscillatory movement?
How can we represent harmonic oscillator motion laws?
What happens to energy in motion harmonic oscillator?
Under the action of which type of force a harmonic oscillatory motion is present?
What is the difference between the damped oscillation and the ideal one?

Thinking about the question

2.I. Kicking life into Classroom: Determining the angular amplitude of oscillation of a swing, trapeze sport, etc.

2.II. Into Lab with InLOT
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material point) with mass m suspended by a nonexpendable wire, with length R (fig.2.1.). If the system is out of equilibrium position (vertical position of the wire) and then left free, it will make oscillatory movements around the equilibrium position. \( \alpha \) angle which between wire at a time and the vertical, is called the angular displacement and \( \alpha_0 \) is angular amplitude. Figure 2.1. shows that the force who tends to restore the system in equilibrium position is:

\[
F = G = m \cdot g \cdot \sin \alpha.
\]

In laboratory reference frame, on oscillation direction the equations of motion, of velocity and acceleration are:

\[
x(t) = A \sin(\omega_0 t - \pi/2), \quad t=0, \quad x=-A \quad (2.1.)
\]

\[
v(t) = \omega_0 A \cos(\omega_0 t - \pi/2) \quad (2.2.)
\]

\[
a(t) = -\omega_0^2 A \sin(\omega_0 t - \pi/2) \quad (2.3.)
\]

note: \( \omega_0 t - \pi/2 = \alpha(t) \), \( A=R \sin \alpha_0 \)

Then, angular displacement follows:

\[
\alpha(t) = \alpha_0 \sin(\omega_0 t - \pi/2), \quad t=0, \quad \alpha=-\alpha_0 \quad (2.4.)
\]

\[
\dot{\alpha}(t) = \omega_0 \alpha_0 \cos(\omega_0 t - \pi/2) \quad (2.5.)
\]

\[
\ddot{\alpha}(t) = -\omega_0^2 \alpha_0 \sin(\omega_0 t - \pi/2) \quad (2.6.)
\]

equations that are adapted of the harmonic oscillations \( (\alpha_0 < 5^\circ) \) of the gravitational pendulum.

**acceleration of gravitational pendulum method**

4. For \( \alpha = 0 \) follows \( a = 0 \); collecting data by InLOT follows \( \Delta t \) between two moments with \( \alpha = 0 \), then follows \( T/2 \) (half period); follows \( T = 2 \Delta t \).

5. in \( x = \pm A \), follows \( \alpha = \pm \alpha_0 \)

6. \( a(2k\pi) = \omega_0^2 A \), and in harmonical oscillations approximation, means \( \alpha_0 < 5^\circ \), follows \( \sin \alpha \approx \alpha \) (radians), we can take into account that the object motion is oriented along Ox axis, and the force which restore the equilibrium is:

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F = G = -mg \alpha = -mg \frac{x}{R} = -kx \quad (2.7)
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\omega_0 = \sqrt{\frac{g}{R} \Rightarrow \omega_0^2 = \frac{g}{R}} \quad (2.9)
\]

\[
\Rightarrow \left( \frac{2\pi}{T} \right)^2 = \frac{g}{R} \quad (2.10)
\]

2.10. expression allows us to calculate the gravitational acceleration with the gravitational pendulum method.

**Obs.:** Is recommended to compare the value obtained with this method and the value obtained using AM in rest on the horizontal surface, \( g_{static} \). Deviation of the value of \( g_{static} \) and \( g_{dynamic} \)’s value is determined by the deviation from the ideal model gravitational pendulum oscillations in harmonic approximation.

2.11.B The determination of amplitude and angular amplitude of the harmonical oscillations

\[
a(2k\pi) = g \frac{A}{R} \Rightarrow A=R \frac{a(2k\pi)}{g} \quad (2.11)
\]

on the other hand:

\[
- \quad a(2k\pi) = a_{max} \quad (from \ measurements) \quad (2.12)
\]

\[
- \quad a((2k+1)\pi) = a_{min} \quad (2.13)
\]

\[
A=R \frac{a_{max}}{g} \Leftrightarrow \alpha_0 = \frac{a_{max}}{g} \quad (2.14)
\]
Materials needed

- InLOT system
- PC
- kit physics: mechanical oscillations
- Worksheet

Safety

Follow the rules of labour protection in the physics laboratory.

Investigation

Name and surname of the participants: 1. __________ , 2. __________ , 3. __________ , 4. __________ , 5. __________

Category □ student; □ teacher; □ athlete; □ other
Age: __________, gender: □ M, □ F

<table>
<thead>
<tr>
<th>Experimental determinations</th>
<th>Action plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravitational acceleration value of the place is: ( g \text{\scriptsize{\text{static}}} = ) \boxed{\text{___________ m/s}^2}</td>
<td>1. Place the accelerometer on a horizontal plane and measure the value of local gravitational acceleration ( g \text{\scriptsize{\text{static}}} ).</td>
</tr>
<tr>
<td>Value component of acceleration perpendicular to the plane (the reference axis Oz of AM) test object, determined using InLOT platform is: ( a_{\perp} = ) \boxed{\text{___________ m/s}^2}</td>
<td>2. Fit a plane at an angle corresponding uniformly accelerated sliding of the object test</td>
</tr>
<tr>
<td>The value of the acceleration component parallel to the plane (the reference axis Ox of AM) test object determined using InLOT platform is: ( a_{\parallel} = ) \boxed{\text{___________ m/s}^2}</td>
<td>3. Next, measure the component perpendicular to the axis ( (a_{\perp}) ), then find ( \theta ) (according with (1.6)).</td>
</tr>
<tr>
<td>Sliding friction coefficient value is: ( \mu = ) \boxed{}</td>
<td>4. Further leave test object (with accelerometer) to free fall flat (uniformly accelerated); determine ( a_{\parallel} ).</td>
</tr>
<tr>
<td>Angle of friction pair testing object - surface inclined plane is: ( \theta_0 = ) \boxed{}</td>
<td>5. Next determine ( \mu ) (according with (1.7)).</td>
</tr>
<tr>
<td></td>
<td>6. Plane angle is fixed at an angle to the test object falls straight and uniform, ie ( a_{\parallel} = 0 ), related to an angle named friction angle ( \theta_0 ), and this leads, according to relationship (1.7), to (1.8) and determine the coefficient of sliding friction between test object surface and inclined plane surface. Compare this value with that obtained at step 5.</td>
</tr>
<tr>
<td></td>
<td>7. If there is a possibility, it may change the parameters of roughness of two surfaces, to test different areas of contact.</td>
</tr>
</tbody>
</table>

\[ ^2 \text{Use InLOT platform} \]
\[ ^3 \text{Use InLOT platform} \]
Analyze the causes of friction and what impact they had on the outcome of the experiment.

**Further investigation**

1. **Relevance.** Students will reflect and find answers identifying possible practical role of the work done, the benefits of science and technology on life in general, the place of science in society, the social role of researcher.

2. **Connection with the real world.** Students will reflect on the practical character of their project, they will understand the importance of experimental data and the practical benefits of using the results.

**Assessment**

**Gallery Tour:** Students will prepare oral presentations to appropriate audiences, which are accompanied by multimedia presentations, brochures and websites. These products must identify current community needs and resources and provide acceptable solutions. Thus, the task turns into a learning project in support of the community, creating an authentic purpose and making a connection with the real world through community.

**Evaluation criterion:**

1. All team members undertake collaborative activities by completing the steps in processing aid given to them and collect data for one of the roles within the team

2. Each member fulfills the role it has in the team. Team members’ work together to achieve a quality presentation

3. Presentation made meet the recommended structure.

4. Explanation contained in the presentation is enlightening to the public

5. Project presentation is eloquent and enlightening for the audience participating.

6. The manner of presentation is attractive and involving public

7. Team members are open to public questions and formulate answers all questions pertinent to public

8. Introducing the team roles demonstrates that members are knowledgeable in all fields covered by the project.

9. Team members speak out loud, communicates a very clear presentation of content, and establish eye contact with audience.

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